

# PhD Proposal

**Place :** Laboratoire Méthodes Formelles (LMF)  
Ecole Normale Supérieure Paris-Saclay, France  
&  
Laboratoire d'Informatique de l'Ecole Polytechnique (LIX)  
Ecole Polytechnique, France

**Title :** Reasoning about Concurrent Game Structures with Numerical Resources

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**Resource-bounded logics.** Many logical formalisms exist for specifying the strategic behaviour of agents in multi-agent systems, including the alternating-time temporal logic ATL [AHK02] and extensions in which actions may consume or produce resources, see e.g. [BF09, ALNR17]. The logic ATL generalises the temporal logic CTL (see survey material about branching-time temporal logics or ATL in [DGL16]) and has been subject to many extensions, including quantitative ones. The logic  $RB\pm ATL$ , introduced in [ALNR14], is one of such extensions and the model-checking problem has been shown  $2EXPTIME$ -complete in [ABDL18]. Many other extensions exist, and not all of them lead to decidable model-checking problems as undecidability can be sometimes concluded by reduction from undecidable decision problems for counter machines such as for vector addition systems with states (VASS) or for Minsky machines.

**Logics in AI and verification games.** In the recent work [ABDL18], formal relationships have been established between model-checking problems for resource-bounded logics and decision problems for VASS so that new decidability results can be established for logical problems or new complexity characterisations can be inherited from problems on counter machines, see e.g. [JLS15, CJLS17]. Of course, this should not come as a real surprise because resource values and counter values are similar objects and logics based on concurrent game structures have inherently games in the semantics. Moreover, earlier works have already explored the connections with counter machines, either to obtain undecidability results or to get complexity lower bounds.

**Objectives of the PhD thesis** The main objective of the PhD proposal is to investigate the computational properties of resource-bounded logics and fragments, to design optimal decision procedures, possibly based on proof systems, and to study its relationships with computational models such as alternating VASS and branching VASS in which the computations are tree-like structures.

Let us provide more details. After becoming familiar with the major resource-bounded logics extending ATL and with the most standard games on VASS as well as the main results about alternating VASS and branching VASS, the candidate will tackle the following problems.

- (1) To identify fragments of the expressive logic  $RB\pm ATL^*$  [ABDL18] for which the current decision procedures for solving the model-checking for  $RB\pm ATL$  or for  $RB\pm ATL^*$  could be drastically simplified. The ultimate goal is to design expressive fragments of  $RB\pm ATL^*$  but restrictive enough to admit relatively low complexity.

- (2) To design VASS games whose main decision problems are equivalent to model-checking problems based on resource-bounded logics, extending the approach presented in [ABDL18].
- (3) In [ABDL18], apart from model-checking, problems related to the synthesis of parameters are introduced in which the formulae contain free variables and the main task is to identify the value of the parameters that make true the formula at hand. For positive formulas, because of monotonicity, this amounts to computing minimal values (known as the Pareto frontier) and the results in [ABDL18] are based on results about decision problems for alternating VASS from [CS14, AMSS13]. So far, the computation of the Pareto frontier for parameterised formulae from the parameterised version of  $RB\pm ATL$  is computationally expensive and the main goal in this part is to identify conditions to extract parameter values with a reasonable computational complexity. We aim at finding a good compromise between complexity and expressivity, possibly restricting the class of concurrent game structures, the class of formulae or the very definition of the satisfaction relation (numerous options are possible).
- (4) Alternating VASS (see e.g. [AMSS13, CS14]) are related to the model-checking problem for resource-bounded logics [ABDL18]. Besides, branching VASS [VGL05, DJLL13, GHLT16, FLL<sup>+</sup>17, Str19] have been quite studied recently and can be viewed as variants of alternating VASS but still having a notion of computation that involves trees. In this part of the project, we aim at importing results for resource-bounded logics to such VASS extensions and alternatively, to obtain new results on AVASS or BVASS that can be used to improve the design of decision procedures dedicated to the model-checking problem for resource-bounded logics. As BVASS are also closely related to the proof theory of the linear logic MELL, see e.g. [dGGS04, Str19], this part of the project will lead to proof-theoretical investigations. Indeed, we may explore the systematic design of proof systems for resource-bounded logics, for instance with sequents or nested sequents, following the methodologies developed in [BS09, ADS15].

**Environment.** The primary research unit will be Laboratoire Méthodes Formelles (LMF) created in 2021. The secondary research unit will be the Laboratoire d’Informatique de l’Ecole Polytechnique (LIX), 10 minutes away from LMF. This new location will facilitate the exchanges between the PhD student and the two supervisors. LMF is a joint laboratory of the CNRS (Centre National de la Recherche Scientifique), ENS Paris-Saclay (Ecole Normale Supérieure Paris-Saclay) and Université Paris-Saclay. LIX (Laboratoire d’Informatique de l’Ecole Polytechnique) is a joint laboratory of the CNRS (Centre National de la Recherche Scientifique) and Ecole Polytechnique. It is located at Palaiseau on Ecole Polytechnique campus.

**Organisation.** The PhD student will be co-supervised by S. Demri (LMF) and by L. Straßburger (LIX), with a standard working environment at LIX and at LMF. By default, the PhD student will spend four days at LMF and one at LIX per week, but this will be flexible depending on the research agenda. The two supervisors are local responsables of a new Stic-Amsud project “Dynamic Logics: Model Theory, Proof Theory and Computational Complexity” (started in 2020 but not actively due to covid-19) with colleagues from Argentina and Brazil. Though, strictly speaking the PhD subject and the agenda of the project are disjoint, one can find common ingredients such as logics for knowledge representation, computational complexity or the design of proof systems, to quote a few keywords. So, the PhD student will have also the opportunity to benefit from the international network of the supervisors.

**Application and starting date.** The starting date of the PhD position should be before Oct. 1st 2021. The candidate must hold a Master degree in Computer Science, with a solid background in Theoretical Computer Science or Symbolic AI. Typically, candidates with a good knowledge on formal methods, logics for artificial intelligence and proof theory are much appreciated. Applications can be sent either in English or in French. To apply to the position, please send a CV to `demri@lsv.fr` and `lutz@lix.polytechnique.fr`.

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